A composite constitutive model for methane hydrate-bearing soils using equivalent granular void ratio

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Introduction

Methane hydrate, a crystalline material made up of a water cage surrounding a methane molecule, forms naturally in regions of permafrost and continental margin sediment where there are appropriate pressure and temperature conditions and sufficient methane gas. The large amount of reserves of methane hydrate has prompted it to be considered as a potential fuel for the future. During the production of methane from hydrates, hydrate dissociation may induce a variety of geological disasters. (Mienert *et al.* 2005). Thus, it is important to simulate the mechanical behavior of methane hydrate-bearing soils (MHBS) for safe extraction of methane from hydrate reservoirs.

In this study, the concept of equivalent granular void ratio is first presented and its rationality in capturing the behavior of MHBS is verified through the reported experimental data. Then a new composite constitutive model using equivalent granular void ratio as a state variable is introduced. The proposed composite model consists of (1) a generalized plasticity model for the behavior of the uncemented granular skeleton and (2) an elastic damage model for the behavior of the hydrate bonds.

Equivalent granular void ratio

Void ratio (e) is commonly used as a state variable in critical state soil mechanics for the interpretation of soil behavior. However, past studies have shown that an addition of small amount of fines in sand decreased its void ratio but the stress-strain behavior essentially remained the same. To properly describe the mechanical behavior of sand-fines mixtures, Thevanayagam *et al.* (2002) proposed an equivalent granular void ratio (e^*) accounting for the 'actual' density of sand-fines mixtures, which is defined as

$$e^* = \frac{e + (1-b)f_c}{1 - (1-b)f} \tag{1}$$

where f_c is the fines content, and b is the fraction of fines which actively take part in the force chain of the sand skeleton.

Microscopic investigation showed that the average particle size of hydrates is about 0.04 mm. Through the results of core drilling, the average particle sizes of the coarse-grained sediments are 0.2 mm extracted from Nankai Trough, or 5 times larger than that of hydrates. Therefore, ignoring the cementing effect of hydrates, it is reasonable to consider the hydrates in MHBS as fines. As a result, it is postulated that equivalent granular void ratio may be adopted as a state variable to describe the stress-strain behavior of MHBS. Eq. (1) is modified as follows:

$$e^* = \frac{e(1 - \chi S_h)}{1 + \chi e S_h}$$
(2)

where S_h is the hydrate saturation and χ is the fraction of hydrates actively participating in the force chain of MHBS.

Model formulation

In this study, the mechanical behavior of MHBS is modeled as a composite material of the uncemented granular skeleton and hydrate bonds (Chazallon and Hicher 1995). The uncemented granular skeleton contains the coarse soil grains as well as the hydrates formed in the intergranular pore spaces resulting from pore-filling and load-bearing accumulation habits. On the other hand the hydrate bonds represent the cementing accumulation habit of hydrates formed at the intergranular contacts. The corresponding stress exhibited on MHBS (σ) is the summation of the stresses of the uncemented

granular skeleton (σ_s) and hydrate bonds (σ_b), written as follows

 $\boldsymbol{\sigma} = \boldsymbol{\sigma}_s + \boldsymbol{\sigma}_b$

In this composite model, the framework of generalized plasticity is adopted for the constitutive modeling of the mechanical behavior of the uncemented granular skeleton. In the generalized plasticity, the stress increment vector ($d\sigma_s$) and strain increment vector ($d\varepsilon$) are related through an elastoplastic matrix D_{ep}^{s} , expressed as follows:

$$d\boldsymbol{\sigma}_{s} = \boldsymbol{D}_{ep}^{s} d\boldsymbol{\varepsilon}$$
(4)

Similar to the classical elastoplasticity, the elastoplastic matrix under generalized plasticity is expressed as follows:

$$\boldsymbol{D}_{ep}^{s} = \boldsymbol{D}_{e}^{s} - \frac{\boldsymbol{D}_{e}^{s} \boldsymbol{n}_{g} \boldsymbol{n}_{f}^{T} \boldsymbol{D}_{e}^{s}}{H + \boldsymbol{n}_{f}^{T} \boldsymbol{D}_{e}^{s} \boldsymbol{n}_{g}}$$
(5)

where D_e^s , H, n_f and n_g are elastic matrix, plastic modulus, loading direction vector, and plastic flow direction vector, respectively.

An elastic law with damage was adopted to model the behavior of the hydrate bonds. According to the elasticity, the stress-strain relationship of the hydrate bonds is written as

$$\boldsymbol{\sigma}_{b} = \boldsymbol{D}_{e}^{b} \boldsymbol{\varepsilon}$$
(6)

where D_e^b is the elastic matrix. Considering the degradation of stiffness with shear deformation, the increment stress-stain relationship for the hydrate bonds is written as

$$d\boldsymbol{\sigma}_{b} = \boldsymbol{D}_{e}^{b} d\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon} d\boldsymbol{D}_{e}^{b}$$
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Model verification

The set of experimental data used here for the model verification came from drained triaxial compression tests conducted by Hyodo et al. (2013), which are currently the most complete set of triaxial tests published on synthesized MHBS specimens. Model simulations of MHBS are presented in Fig. 1. By comparisons, the $\varepsilon_a \square q$ and $\varepsilon_a \square \varepsilon_v$ relations predicted by the model can well describe the overall trend of the deformation process observed in the tests for MHBS. The increase in stiffness, peak strength and dilatancy with increasing hydrate saturation can be well captured through model



Figure 1. Comparison between drained triaxial test results and model simulations for MHBS with varying hydrate saturations (test data from Hyodo et al. 2013)

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