## A Bayesian hierarchical framework for induced seismicity hazard associated with deep underground fluid injection

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## Introduction

In the last years, a significant increase in seismicity was caused by deep fluid injections. Fluid-induced seismicity has characteristics that distinguish it from natural seismicity. In particular, the seismic rate is a consequence of an interaction between time-variant fluid injection and physical properties, which are site-dependent. One notable difference between natural and induced seismicity is that both hazard and risk are time-dependent. In this talk, we present a non-uniform Poisson process to model induced seismicity associated with deep underground fluid injection. The time-variant rate of the Poissonian process is described as a function of the fluid-injection rate and a set of physical parameters defining the ground characteristics. The set of parameters are considered as random variables, and their uncertainty reflects the source-to-source variability. A significant strength of the Bayesian approach is that it allows uncertainties and expert judgments about the ground parameters to be encoded into a joint prior distribution of the model parameters. Moreover, as soon as the project starts and physical information become available, the Bayesian framework allows for the computation of the posterior distribution for the ground parameters, and the formulation of predictive models. After presenting the updating rules, the talk is concluded by introducing a forecast model for predicting the number and the magnitude of induced events for a given future time frame.

## Probabilistic and forecasting model

In this talk, we propose a Non-Homogeneous Poisson Process (NHPP) to model the time occurrence of fluid induced seismic event. NHPP models are completely characterized by a time varying rate,  $\lambda(t; \theta)$ , which in this context is defined as (Mignan *et al.* 2017, Broccardo *et al.* 2017)

$$\lambda(t|\boldsymbol{\theta}) = \begin{cases} 10^{a_{fb}-bm_0} \dot{V}(t) & ;t \le t_{shut-in} \\ 10^{a_{fb}-b_{fb}} \dot{V}(t_{shut-in}) \exp\left(-\frac{t-t_{shut-in}}{\tau}\right) & ;t > t_{shut-in} \end{cases}$$
(1)

where  $\dot{V}(t)$  is the injection flow rate;  $\boldsymbol{\theta} = [a_{fb}, b_{fb}, \tau]$  is the set of parameters describing activation feedback, the earthquake size ratio, and mean relaxation time; moreover,  $m_0$  is the magnitude of completeness, and  $t_s$  the shutin time. In the proposed Bayesian setting, we consider the parameter of the model as a random vector,  $\boldsymbol{\theta} = [A_{fb}, B_{fb}, T]$  to encode source-to-source variability. We define the prior joint-probability distribution as  $f'_{\theta}(a_{fb}, b_{fb}, \tau) = f'_{A_{fb}}(a_{fb})f'_{B_{fb}}(b_{fb})f'_{T}(\tau)$ . Despite assuming independence in this definition, the posterior distribution encrypts any type of correlation structures emerging from the data. The marginal prior distributions are selected as follow:  $f'_{A_{fb}}(a_{fb}) = \text{Beta}(a_{fb}; \theta_{a_{fb}}), f'_{B_{fb}}(a_{fb}) = \text{Beta}(b_{fb}; \theta_{b_{fb}}), f'_{T}(\tau) = \text{Gamma}(\tau; \theta_{\tau})$ , where  $\theta_{a_{fb}}, \theta_{b_{fb}}, \theta_{\tau}$  are the model parameters of the prior distributions. These parameters are determined via Maximum Likelihood Estimate (MLE) on six datasets (Mignan *et al.* 2017, Broccardo *et al.* 2017). Given a set of observations  $\mathcal{D}(t) = [t_1, ..., t_n, ..., t_N; m_1, ..., m_N, ..., m_N]$ , up to a time t, where  $t_n$  is a given occurrence time and  $m_n$  a magnitude event with  $t_N < t$ , the posterior probability distribution of the model parameters,  $f''_{\theta}(\theta|\mathcal{D}(t))$ , is updated as  $f''_{\theta}(\theta|\mathcal{D}(t)) = c(t)\mathcal{L}(\theta|\mathcal{D}(t))f'_{\theta}(\theta)$ , where  $\mathcal{L}(\theta|\mathcal{D})$  is the likelihood function, and c(t) is a normalizing factor. Given a magnitude frequency distribution,  $f_M(m|b_{bf})$ , the likelihood function is defined as follows (Broccardo *et al.* 2017)

$$\mathcal{L}(\boldsymbol{\theta}|\mathcal{D}(t)) = \left[\prod_{n=1}^{N} \lambda(t_n|\boldsymbol{\theta}) f_M(m_n|b_{bf})\right] \exp[-\Lambda(t|\boldsymbol{\theta})].$$
(2)

Once the posterior distribution is computed, the predictive model for the number of event in a given time windows [t, t + h] can be written as follows:

$$P(N(t) = n|\mathcal{D}(t)) = \int_{\theta} \left[ \frac{\left( \int_{t}^{t+h} \lambda(t'|\boldsymbol{\theta}) dt' \right)^{n}}{n!} \exp\left[ -\int_{t}^{t+h} \lambda(t'|\boldsymbol{\theta}) dt' \right] \right] f''_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\mathcal{D}(t)) d\boldsymbol{\theta}.$$
(3)

and the probability of the Maximum Magnitude  $M_{max}$  as

$$P(M_{max} > m | \mathcal{D}(t)) = 1 - \int_{\boldsymbol{b}} \left[ \sum_{n=1}^{\infty} P(M > m | \boldsymbol{b})^n P(N = n | \mathcal{D}(t)) \right] f''_{\boldsymbol{\theta}}(\boldsymbol{b} | \mathcal{D}(t)) d\boldsymbol{b}.$$

$$\tag{4}$$

Figure 4 shows a short-term forecasting model based on the predictive models of Eq. (3) and Eq. (4), for the Basel 2006 fluid-induced seismicity sequence (Broccardo *et al.* 2017). Specifically, it is shown the prediction of the number of events N(t) for a h = 4[hours] time interval together with a 90% credible interval. In addition, it is reported the prediction of  $M_{max}$  (maximum magnitude) for a h = 4[hours] time interval together with a 90% asymmetric credible interval. Figure 4 suggests that the proposed forecast model accurately predicts both the number and maximum magnitude of events for the given time window. Finally, by further defining a decision-making criterion, the credible intervals clearly become an important potential tool for defining a mitigation strategy.



Fig. 1: Forecasting model for Basel 2006. a) Predictive model for N(t), red bars are 90% credible intervals, black dot the observed number of events; b) Probability density distribution of N(t), grey dashed lines 90% credible interval; c) Time series of magnitude events, red bar asymmetric credible interval for  $M_{max}$  in a 4[hours] time window, yellow/red stems observed event, grey stems past events; d)  $f(m, t + h|\mathcal{D}(t))$  red area asymmetric credible interval; e) Full predictive model for N(t); d) Full prediction for  $M_{max}$ . Source: Broccardo et al. (2017)

## References

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