A Thermo-Hydro-Mechanical Model for Soil Freezing/Thawing

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Model description

This contribution presents an extended soil freezing and thawing model capable of simulating all important thermo-hydro-mechanical phenomena occurring in freezing and thawing in a saturated soil constituting three phases: solid matrix, liquid water and ice. The model includes phase change, fluid mass transport, convective-conductive heat flow, solid phase deformation, melting point depression, unfrozen water content and cryogenic suction. The solid phase is assumed temperature dependent elastic. The model utilizes the averaging theory for describing the balance equations, as follows (Lewis & Schrefler, 1998):

Momentum balance

$$\nabla \cdot \left[\mathbf{D}_s \left(\mathbf{L} \frac{\partial \mathbf{u}}{\partial t} - \frac{1}{3} \mathbf{m} \beta_s \frac{\partial T_s}{\partial t} \right) - \mathbf{m} \alpha \frac{\partial p_s}{\partial t} \right] + \frac{\partial \rho_{eff}}{\partial t} \mathbf{g} = 0$$

where $\mathbf{u}$ is the displacement vector, T_s is the porous matrix temperature; $\mathbf{D}_s$, β_s and α are the solid stiffness, volumetric thermal expansion coefficient and Biot's coefficient, respectively; $\mathbf{g}$ is the gravitational vector; $\mathbf{L}$ is a differential operator ($\equiv \partial/\partial x_i$), $\mathbf{m} = [1,1,1,0,0,0]^T$, ρ_{eff} is the effective density of multiphase medium, and p_s is the pressure that is exerted on the solid matrix from the pore water.

Mass balance

$$\rho_m \frac{\partial \varphi}{\partial t} + \varphi \frac{\partial \rho_m}{\partial t} + \nabla \cdot \left(\rho_{lw} \mathbf{v}_{lw} \right) + \varphi \rho_m \mathbf{m}^T \mathbf{L} \frac{\partial \mathbf{u}}{\partial t} = 0$$

in which ρ_m is the mass density of ice-liquid water-mixture, φ is the porosity, ρ_{lw} is the mass density of liquid water, and $\mathbf{v}_{lw}$ is the mass averaged velocity of liquid water.

Energy balance

$$\frac{\partial}{\partial t} \left[(1-\varphi) \rho_s h_s + \varphi \rho_m h_m \right] + \nabla \cdot \left(\rho_{lw} h_{lw} \mathbf{v}_{lw} \right) + \nabla \cdot \left(-\boldsymbol{\lambda}_{eff} \cdot \nabla T_s \right) = 0$$

in which ρ_s is the mass density of solid, h_m is water mixture specific enthalpy, h_s is solid specific enthalpy, h_{lw} is the specific enthalpy of liquid water, and λ_{eff} is the effective thermal conductivity of the multiphase medium.

The constitutive relationship for water is based on equation of state (EOS), which has been established by combining thermodynamic data of subcooled liquid water, supercooled liquid water, and ice.

The soil freezing characteristic curve (SFCC) is described via an empirical relationship as follows:

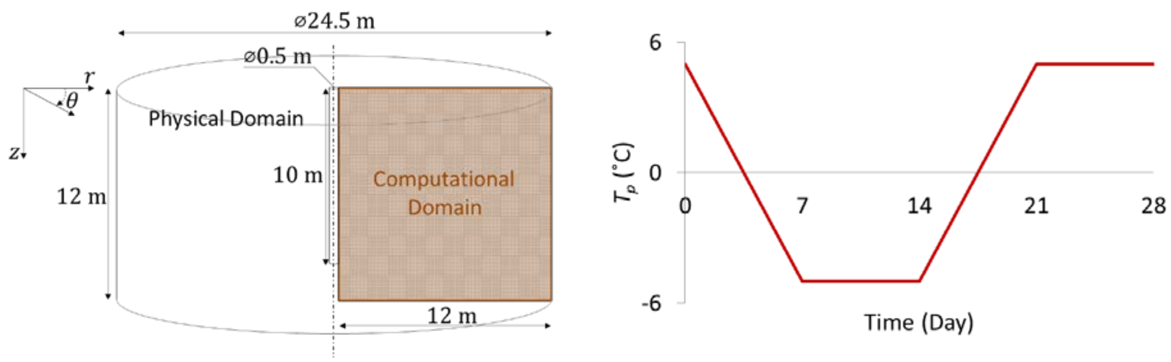
$$S_{lw} = S^* + \left(1 - S^* \right) e^{a \left(T_w - T_f \right)}$$

in which S^* is the residual unfrozen water content at relatively cold condition, T_w is water temperature, T_f is the bulk freezing temperature, and a is a material constant.

The governing equations are solved using a mixed discretization scheme (Arzanfudi & Al-Khoury, 2017) in which the extended finite element (XFEM) is utilized to model the cryogenic suction.

Numerical example

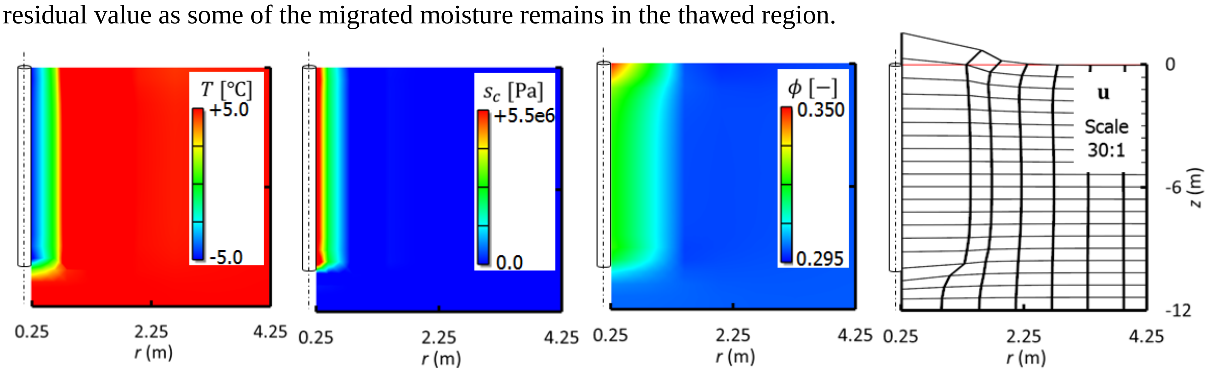
A numerical example demonstrating the capabilities of the model to simulate freezing and thawing in soil is presented. The geometry is designated to resemble a soil mass surrounding an energy pile as shown in Fig. 1a. For soil, a density of 1600 $\mathrm{kg/m^3}$, a permeability of 1 mD, a porosity of 0.3, a heat capacity of 900 J $\mathrm{kg^{-1}K^{-1}}$, a thermal conductivity of 1 $\mathrm{Wm^{-2}K^{-1}}$, and a thermal expansion of $5 \times 10^{-6}\ \mathrm{K^{-1}}$ are assumed. The soil elastic modulus, E, is assumed temperature dependent; $E(T) = E_0 e^{-b(T_s - T_f)}$, with $E_0 = 5.0$ MPa and $b = 0.1\ \mathrm{K^{-1}}$. The heat source represents an energy pile coinciding along the symmetrical axis of the soil, of which the temperature varies as shown in Fig. 1b.



(a) (b)

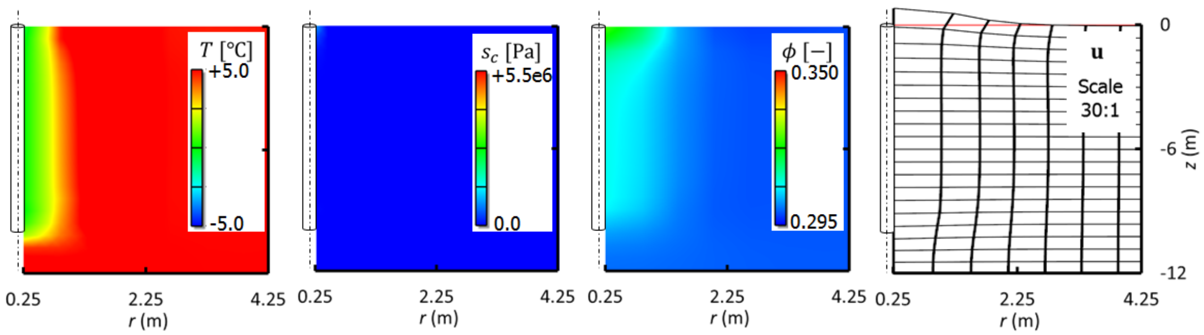
Fig. 1: Geometry and loading: (a) physical and computational domains and (b) heat boundary condition.

The finite element domain is discretized using 400 linear quadrilateral axisymmetric finite elements. Figs. 2 and 3 show the computational results of the temperature, cryogenic suction, porosity expansion and solid matrix deformation at $t = 14\ days$ where maximum freezing occurs, and at $t = 19\ days$, just after thawing, respectively. Fig. 2b shows that upon freezing, the cryogenic suction arises, associated with porosity expansion and solid matrix heaving, Figs. 2c and 2d. Fig. 3 shows that after thawing, the cryogenic suction disappears, and the porosity and heaving are decreased. Although the solid phase is elastic, the porosity and heaving exhibit some residual value as some of the migrated moisture remains in the thawed region.



(a) (b) (c) (d)

Fig. 2: Computational results in the freezing condition (t = 14 days): (a) temperature, (b) cryogenic suction, (c) porosities, and (d) deformations.



(a) (b) (c) (d)

Fig. 3: Computational results in the thawing condition (t = 19 days): (a) temperature, (b) cryogenic suction, (c) porosities, and (d) deformations.

References

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