Role of Reynolds Number in Two-Phase Fluid Flow

Dong Hun Kang¹, Tae Sup Yun^{1*}

¹ Department of Civil and Environmental Engineering, Yonsei University, Seoul, Korea * taesup@yonsei.ac.kr

Darcy-Forchheimer flow boundary

The dimesionless numbers, Capillary Number (*Ca*) and Viscosity Ratio (*M*), are primary parameters to determine the invasion pattern and displacement efficiency in a wide range of applications such as contaminant flow, methane production from hydrate-bearing sediments, hydrocarbon recovery, and greenhouse gas sequestration (Zheng et al., 2017). Yet, these two values do not represent the effect of fluid density and pore size such that it is highly desired to consider Reynolds Number (*Re*) in two-phase fluid flow. The inertia effect has been introduced by Darcy-Forchheimer equation in the single-phase fluid flow whereas it is difficult to outline the inertia force dominant regime for two-phase fluid flow in porous media because both capillary and viscous force are dominant. The modified friction factor is a measure of inertia effect as follows (Liu et al., 1994);

$$f_M = f_F Re = -\nabla p \frac{8\phi^3}{q \,\mu s^2 T}$$

The single-phase fluid flow simulation using 2D microfluidic domain by the lattice Boltzmann method allows computing the permeability and f_M with varying Reynolds number as shown in Fig.1 Both values are constant up to log*Re* ~ 1 and become nonlinear. The critical Reynolds number outlinging the boundary line between Darcy and Forchheimer regimes is given by log*Re*_c = 1.25 (Zeng and Grigg, 2006).



Fig. 1: 2D microfluidic domain and computed values of permeability and modified friction factor

Invasion of non-wetting fluid and transitional boundary

We simulate the two-phase fluid flow by using Rothman-Keller model (Kang and Yun, 2018). The dimensionless numbers are set as follows; Reynolds number $\log Re = -2$ to 2, the constant Capillary number $\log Ca = -5$, and three viscosity ratio $\log M = -2$, -1, and 0. The constant flow rate is applied to the inlet and the constant pressure is imposed to the outlet. As the multiple preferential flow paths are developed, the advanced fingering advances toward the outlet while the following fingering tends to remain immobile with increasing flow pressure. Once the advanced fingering is discharged, the flow pressure in the following fingering becomes dissipated and begins moving. Also, results show that the non-wetting fluid readily displaces the wetting fluid with increasing Re and M and the main streamline become more tortuous at high M (Fig.2).

We determine the residual satuation values of non-wetting fluid at equilibrium state and plot them with $\log Re$ in Figure 2. It is apparent that there is the transitional boundary where the residual saturation values becomes fluctuated depending on $\log Re$ and $\log M$. For $\log M = 0$ cases, the residual saturation value of ~ 0.75 becomes constant up to $\log Re \sim -1$. Then, the slight concave shape of saturation exists followed by the constant residual saturation again. As the viscosity ratio decreases (e.g., $\log M = -1$ and $\log M = -2$), the transitional boundary where the residual saturation becomes non-constant is shifted to the high value of Reynolds number. In other words, the inertia-negligible regimes is extended up to $\log Re = 0.5$ at $\log M = -2$ case in terms of 'residual saturation'. Although the previous experimental studies provided the quantitative measure of residual saturation in carbon dioxide replacement with varying *Ca* and *M*, those values reside within the transitional boundary between the inertia-negligible and inertia-effective regimes. It is noted that the variation of residual saturation in this transitional regime is approximately $10 \sim 20\%$ such that the inertia effect may dominate over the capillary and viscous forces therein.



Fig. 2: Invasion patterns and the boundary of inertia effective regime

As the two-phase fluid flow inherently produces the main streamline, it is doable to adopt the modified friction factor introduced earlier and to use the morphological features extracted from non-wetting fluid invasion (i.e., volume fraction of main flow paths, effective tortuosity, surface area of invading fluid). The modified friction factor for two-phase fluid flow is computed by considering the geometry of main flow paths and is plotted with Re for different viscosity ratio (Fig.2). It emphasizes that the critical Reynolds number, $logRe_c = 1.25$ still outlines the inertia-force dominant regime and the entire evolution of f_M is similar to the case for single-phase fluid flow (red line). It is noted that the average pore size in carbon dioxide sequestration ranges from 10^{-4} to 1 mm such that the absolute size and fluid density effect prevail in experimental and numerical studies. The flow in low*Re* condition may be governed by Darcy flow while the increase of Reynolds number adds the inertia effective term. Therefore, it is critical to carefully consider which regime the simulation resides in evaluating the invasion patterns and residual saturation.

References

- Kang, DH, Yun TS, Kim KY, Jang, J (2016). Effect of hydrate nucleation mechanisms and capillarity on permeability reduction in granular media. Geophys. Res. Lett. 43: 9018–9025. doi.org/10.1002/2016GL070511
- Liu S, Afacan A, Masliyah J (1994) Steady incompressible laminar flow in porous media. Chem. Eng. Sci. 49: 3565–3586. doi.org/10.1016/0009-2509(94)00168-5

Zeng Z, Grigg R (2006) A criterion for non-Darcy flow in porous media. Transp. Porous Media 63: 57–69. doi.org/10.1007/s11242-005-2720-3

Zheng X, Mahabadi N, Yun TS, Jang J (2017) Effect of capillary and viscous force on CO₂ saturation and invasion pattern in the microfluidic chip. J. Geophys. Res. Solid Earth 122, 1634–1647. doi.org/10.1002/2016JB013908