# Using Biot's coefficient in estimation of thermal conductivity of sandstones

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## Introduction

In the subsurface industry, assessing depth variations of key reservoir properties is often limited to estimation from downhole logging campaigns, as core material is minimal. Thermal conductivity is in geothermal engineering one key property often derived using logging data combined with predictions of empirical or theoretical character. Porosity, $\phi$, is one of the key parameters classifying porous rocks and are in theoretical models of thermal conductivity, $\lambda$, used to quantify the volume of solid and fluid constituent. Models solely based on porosity such as the conventional geometric mean defined as $\lambda = \lambda_f^{\phi} \lambda_s^{1-\phi}$ where $\lambda_f$ and $\lambda_s$ are thermal conductivity of fluid and solid constituents respectively, however only captures little of the rock texture. As thermal conductivity of solid constituents is typically orders of magnitude larger than that of fluids it conjointly with the texture of the solid constituent governs the overall heat transfer. Thus, a theoretical model based on parameters describing the rock texture is essential of modelling of the thermal conductivity. By exclusively utilizing para-meters derived through conventional interpretation of standard logs, quantifying the rock texture of sandstones, we constructed a simplified rock model in a unit volume and a conjoining theoretical model for prediction of thermal conductivity provided knowledge of the mineralogy from e.g. drill cuttings. For model validation, we compared measured thermal conductivity to model predictions using laboratory and log data from the Gassum sandstone formation and we find that the new model gives a better prediction as compared to the conventional the geometric mean.

## *Theory*

In the simplified unit rock volume with porosity $\phi$, we divided the volume of solid constituents $(1-\phi)$ in 1) a load bearing fraction equal to $(1-\alpha)$ where $\alpha$ is Biot's coefficient (Biot, 1941) and $(1-\alpha)$ conceptually quantifies the grain to grain contact area, 2) non-load bearing solids, e.g. clay minerals suspended in the pore space and with a total volume of $V_{sus}$ and 3) the residual solid volume $(\alpha - \phi - V_{sus})$. The total porosity is divided in 1) the pore volume open for fluid flow in one direction, $c_M\phi$, are quantified by the factor, $c_M$, derived from porosity after Mortensen et al. (1998) and physical consideration identical to that of Kozeny's factor for permeability and 2) the residual pore volume $(1-c_M)\phi$. We propose that a one-dimensional heat flow passes through constituents in three parallel heat paths: solid, fluid, and solid-fluid in series. Cross sections of the single constituents is quantified by $(1-\alpha)$ and $(1-c_M)\phi$ respectively. The mixed constituent heat path consist of the residuals and the volume of non-load bearing solids arranged in series. Thus, we utilize the conceptual ideas that the grain contact area, quantified through Biot's coefficient equally governs heat transfer and material stiffness and the pore space open for fluid flow in one direction equally governs heat transfer in the same direction. Summarizing contributions to the overall thermal conductivity, $\lambda$, from each heat path equals

$$\lambda = (1-\alpha)\lambda_{lbs} + c_M\phi\lambda_f + (\alpha - c_M\phi)^2 \left( \frac{(1-c_M)\phi}{\lambda_f} + \frac{V_{sus}}{\lambda_{sus}} + \frac{\alpha-\phi-V_{sus}}{\lambda_{lbs}} \right)^{-1}, c_M = \left\{ 4\cos\left[ \frac{1}{3}\cos^{-1}\left( \phi\frac{64}{\pi^3} - 1 \right) + \frac{4\pi}{3} \right] + 4 \right\}^{-1}$$

where $\lambda_{lbs}$, $\lambda_{sus}$, $\lambda_f$ are thermal conductivities of the load bearing solid, the suspended non-load bearing solid, and the saturating fluid respectively. Biot's coefficient is, provided knowledge of the mineral bulk modulus, $K_{min}$ defined as $\alpha = 1 - K_{dra}/K_{min}$ where $K_{dra}$ is the drained bulk modulus typically approximated through fluid substitution of saturated modulus derived from saturated density and ultrasonic velocities.

## *Method and Materials*

Logging data and corresponding core material from the depth range of 1600 to 1700 meters of an exploration well located on mid Zealand near Stenlille, Denmark were used for model validation. The depth interval represents the quartz dominated Gassum sandstone Formation and data includes bulk density, electrical resistivity, natural

Gamma Ray (GR) and compressional wave velocity. Using conventional log interpretation, we derived $\phi$, $V_{sus}$. As shear wave velocities are unavailable we approximated Biot's coefficient as $\delta = 1 - M_{dra}/M_{min}$ with $M_{dra}$ using fluid substitution, density and the compressional wave velocity. For $M_{min}$ we assumed a quartz compressional mineral modulus of 97 GPa. Thermal conductivity, $\lambda_{measured}$, was measured on slabbed cores in the dry state.

## *Results and Discussion*

Using $\lambda_{lbs}$, $\lambda_{sus}$, $\lambda_{f}$ of respectively 7.7, 6.0 and 0.025 $Wm^{-1}K^{-1}$, predictions of dry thermal conductivity from logging data versus measured thermal conductivity show good agreement in the studied Gassum sandstone formation (Fig. 1a). Especially in the sections with low porosity, our model provides a good agreement with experimental results as compared to the geometric mean (Fig. 1a). In terms of Root Mean Square Error (RMSE), model predictions of this work show approximately twice the accuracy of prediction from the geometrical mean (Fig. 1b), justifying the proposed model and its applicability with downhole logging data.

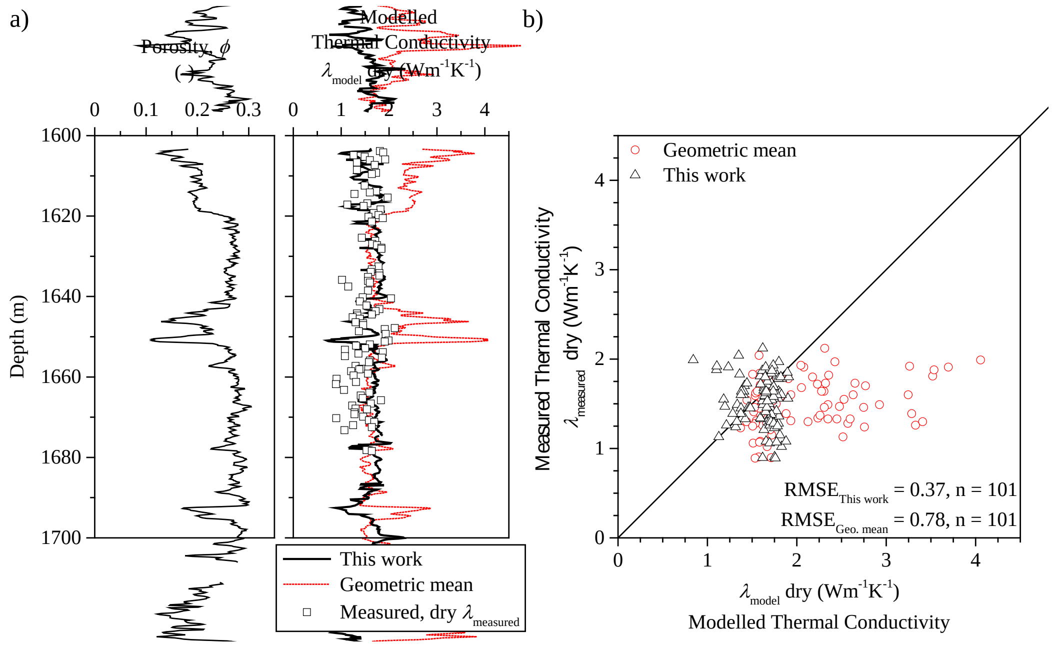


**Fig. 1: a) Porosity and modelled dry thermal conductivity versus depth. The geometric mean is derived using $\lambda_s = \lambda_{lbs}$. b) Measured thermal conductivity versus modelled thermal conductivity on Gassum sandstone. Round and squared markers show modelled results of respectively geometric mean and this work.**

## Conclusion

We validated a new theoretical model of thermal conductivity with application to sandstones by use of logging data. Including quantifications of the heat transfer in cross sections of both solid and fluid constituents derived from measurable parameters, the proposed model is able to predict thermal conductivity with good agreement to experimental results and because input data are derived from physical properties, constraints implicit in semi and empirical relations are removed, thus showing an improvement compared to conventional porosity-based models.

## Acknowledgements

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## *References*

Biot, M.A., 1941, General theory for three-dimensional consolidation: Journal of Applied Physics, 12, 155–164Mortensen, J., Engstrøm, F., and Lind, I. (1998) The relation among porosity, permeability, and the specific surface of chalk from the Gorm field, Danish North Sea. SPE Reservoir Evaluation and Engineering, **3**, 245–251