

Chemo-mechanical Coupling of Subcritical Crack Propagation via Chemo-elasticity

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Proposed Constitutive Model of Chemo-elasticity

A reactive chemo-plasticity model for studying crack propagation by the presence of water is presented in the authors' previous work (Hu and Hueckel, 2013), which features the yield limit of soil/rock depending on the mineral mass removal via dissolution. The rate of dissolution is proportional to variable internal specific surface area, which in turn is assumed a function of dilative plastic deformation. Johnson's approximation is adopted to make involved fields axially symmetric in 2D with respect to the crack tip point, so that semi-analytical solutions can be obtained. However, in this study, the constraints of axi-symmetry are removed by extending the concept of Airy potentials of classical elasticity (Airy, 1863) into the coupled reactive-chemo-elastic domain. The proposed constitutive model of chemo-elasticity describes as

$$\varepsilon_{ij} = K_{ijkl} \cdot \sigma_{kl} + \tilde{\alpha}(\xi - \xi_0)\delta_{ij}; K_{ijkl} = \text{const.}; \tilde{\alpha} = \tilde{\alpha}(\varepsilon_q)$$

where ε_{ij} and σ_{kl} are the total strain and stress, respectively. ξ denotes a scalar variable of integrated mineral mass removal, ξ_0 denoting the background mineral dissolution. The compliance K_{ijkl} is assumed as constant and will be referred to as chemical-elastic compliance, in analogy to the uncoupled thermo-elastic moduli (Nowacki, 1962). $\tilde{\alpha}$ denotes the isotropic chemical shrinking coefficient, which is analogous to a thermal expansion coefficient. However, this chemical shrinking coefficient might not be a constant, but dependent on the level of stress or strain. This is because the mass removal via dissolution depends on the specific surface area of the interface between the solid and fluid phases, which is related to the irreversible deformation/strain that represents micro-cracking. Hence, here we assume that $\tilde{\alpha}$ is a function of the deviatoric strain invariant, which makes it a mechanically coupled chemical shrinking coefficient. Note that compression is defined as positive in this model, and therefore the chemical shrinking coefficient $\tilde{\alpha}$ is positively defined.

Application of the extended Airy Function to Subcritical Crack Propagation

A typical scenario of a plane strain crack subject to chemical dissolution as well as fluid pressurizing on the crack surfaces (due to e.g. chemical injection) is described in Figure 1. A uniformly distributed fluid pressure P_f is assumed acting on the crack surfaces ($\theta = \pm\pi$) and a confining pressure P_b is prescribed, representing overburden pressure. A small cylindrical singularity zone is assumed around the crack tip with a finite prescribed surface traction, P_a at its perimeter at $r = a$ to avoid stress singularity. The chemical processes inside the process zone, including the pathways for solute transport and ion diffusion, are assumed as axisymmetric around the crack tip point for simplicity. The rate of acidity sensitive calcite dissolution is assumed to be a function of local hydrogen concentration and hence the reactive transport process of the solute is coupled with the diffusion of hydrogen.

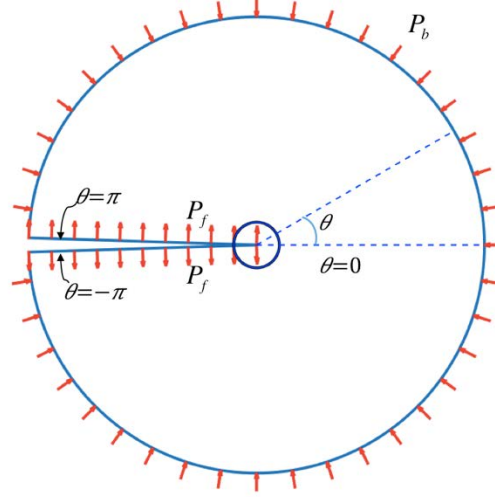


Fig. 1: Sketch of the 2D pressurized crack model. A uniformly distributed fluid pressure P_f is assumed acting on the crack surfaces ($\theta = \pm\pi$) and a confining pressure P_b is prescribed, representing overburden pressure. A small cavity is assumed at the crack tip to avoid stress singularity at $r = 0$

Combining the stress-strain relationship (the proposed chemo-elastic model), the strain compatibility condition (i.e. $\epsilon_{xx,yy} + \epsilon_{yy,xx} = 2\epsilon_{xy,xy}$) and the equilibrium equations, expressed via a scalar Airy potential ϕ , we obtain

$$\nabla^4 \phi + \frac{E}{1-\nu^2} \nabla^2 \{\tilde{\alpha}(r, \theta) \cdot \xi\} = 0$$

The general solution for Airy function is expressed as a sum of two components, the particular solution $\phi^{(p)}$ and the homogeneous solution $\phi^{(h)}$. The particular solution $\phi^{(p)}$ can be obtained by solving a Poisson equation with a prescribed source depending on local chemical shrinkage, while the homogeneous solution $\phi^{(h)}$ is obtained with the method of least-square fitting. Note that the mechanical boundary conditions described in Figure 1 are now re-written as

$$\text{At } r = a, \quad \sigma_r^{(p)} + \sigma_r^{(h)} = P_b; \quad \tau_{r\theta}^{(p)} + \tau_{r\theta}^{(h)} = 0$$

$$\text{At } r = b, \quad \sigma_r^{(p)} + \sigma_r^{(h)} = P_b; \quad \tau_{r\theta}^{(p)} + \tau_{r\theta}^{(h)} = 0$$

$$\text{At } \theta = \pm\pi, \quad \sigma_\theta^{(p)} + \sigma_\theta^{(h)} = P_f; \quad \tau_{r\theta}^{(p)} + \tau_{r\theta}^{(h)} = 0$$

References

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