An effective stress framework for clayey geomaterials

Angelica Tuttolomondo1 *, Alessio Ferrari1, Lyesse Laloui 1

1 École Polytechnique Fédérale de Lausanne (EPFL), School of Architecture, Civil and Environmental Engineering (ENAC), Laboratory for Soil Mechanics (LMS), CH-1015 Lausanne, Switzerland.
* angelica.tuttolomondo@epfl.ch

Introduction

The correct management of nuclear wastes and the preferred option of inserting them into appropriate geological formations raise, among others, challenging questions of geomechanical nature. The engineering solution designed for this purpose involves soils and rocks intended to be subjected to several processes of different physical and chemical nature. In addition, for the suitability of the project to be ensured, the time scale in which the mentioned processes and their effects need to be analysed is very large. Choosing an appropriate modelling framework is the basis for making predictions and engineering decisions of significant importance for current and future society. If the modelling framework chosen involves the use of the effective stress concept, the following steps must be part of the modelling process: (1) assumption of the validity of the effective stress principle; (2) geological and geotechnical characterization of the geomaterial of interest; (3) identification of the relevant physical-chemical phenomena affecting the mechanical behaviour in the problem of interest and in the time scale of the specific engineering application; (4) choice / development of the most effective stress framework for the problem of interest; (5) verification of the consistency between the chosen/developed effective stress framework, the mechanical behaviour of the geomaterial subjected to the identified physical-chemical processes and the validity of the principle of effective stress. The historical development of the Terzaghi’s principle and the main effective stress definitions revised for constitutive modelling purposes can be found in Nuth and Laloui (2008) where the work of reviewing of other authors are also highlighted (Jardine et al., 2004). The main definitions of effective stress for saturated clayey soils, explicating the dependence of mechanical behaviour on physicochemical phenomena, are instead reviewed in Hueckel (1992). The purpose of this work is to develop an effective stress framework for geomaterials with non-negligible clay component involved in engineering applications for which processes inducing changes in electrochemical forces and /or non-zero values of matric suction are considered to be relevant.

Framework development

It is well recognized in the literature that forces of electrochemical nature play an important role in governing the mechanical behaviour of clays. In such materials, the interaction between the liquid phase and the solid phase is made particularly complex due to the significant differences between their atomic structures. The dry clay minerals are characterized by a crystalline structure whose electroneutrality is guaranteed by the presence of cations on the external surface. These cations make possible balancing the lack of positive charge that they should have due to the isomorphic substitution of Si4+ and Al3+ ions by other cations of lower valence. In the presence of water, the cations, previously parts of the minerals, dissociate (Bolt and Bruggenwert, 1976). The liquid phase of the soil, not only will contain the dissociated cations but also ions of dissolved salts coming from different origins. Under the framework of continuum mechanics and thermodynamics, the Bishop's effective stress definition, conventionally applied for unsaturated soils, is in this work reformulated in order to take into account the described chemical-physical specificities of clayey soils. For this purpose, a Representative Elementary Volume (REV) of soil containing a clay component has been thought to be placed in contact with a fluid pressure measurement system. The pressure measurement system contains an aqueous solution whose composition is not affected by the liquid phase-solid phase interaction which characterizes the liquid phase within the soil. The water pressure in the soil is determined by imposing the following conditions: electroneutrality of the geomaterial, electroneutrality of the liquid solution in the measurement system, thermodynamic equilibrium applied to both solutes and water in the soil and in the measurement system. As a consequence, the water pressure within the soil \( u_{w,s} \) can be expressed thanks to the following expression:

\[
U_{w,s} = U_a - S_m + \frac{RT}{V} \ln \frac{x_{w,m}}{x_{w,s}}
\]  

(1)
in which: $u_a$ is the air pressure, $s_m$ is the matric suction (defined as the difference between the air pressure and the measured pressure), $R$ is the gas constant, $T$ the temperature, $\bar{V}$ the molar volume of water, $x_{w,m}$ the molar fraction of water in the measurement system, $x_{w,s}$ the molar fraction of water in the liquid solution within the soil. The term $\frac{RT}{\bar{V}} \ln \frac{x_{w,m}}{x_{w,s}}$ is called effective solute suction and identified with the notation $s_{o,e}$. Replacing the water pressure in the soil in the Bishop’s effective stress expression allows the following expression to be obtained:

$$\sigma'_{ij} = (\sigma_{ij} - u_a \delta_{ij}) + \chi (s_m - s_{o,e}) \delta_{ij}$$

(2)

The developed expression allows taking into account that the mechanical behaviour of a soil with a relevant clay component is not only a function of the net stress $(\sigma_{ij} - u_a \delta_{ij})$ and of the matric suction $(s_m)$ but also of a chemical variable $s_{o,e}$ whose value depends on the soil’s mineralogy and on the liquid phase composition.

**Results and discussion**

By using the possible approximation of $\chi = Sr$, experimental data have been analysed using the Generalized Bishop’s effective stress (Nuth and Laloui, 2008) and the proposed framework. The theoretical development allows obtaining an analytical expression of the effective solute suction as a function of the degree of saturation. A model for describing such curve is proposed. The results obtained by reinterpreting experimental data of clayey geomaterials allow concluding that the introduction of a new variable is justified by the better consistency of the proposed framework, with respect to the existing one, with the principle of effective stress. An example of comparison of interpretation of experimental data is given in Figure 1. In particular, shear strength data and water retention curves have been used in this example for back predicting the $s_{o,e} - Sr$ curve. Focusing on the results obtained by using the Generalized Bishop’s effective stress, the non-linear trend of the retention curve provides a contribution to the effective stress of the type $Sr s_m$ which varies almost linearly with the matric suction. As a consequence, an almost linear variation of the shear strength envelope with respect to the matric suction is gained. The results support the physical interpretation of the developed effective stress and reveal the suitability of the latter for chemo-hydro-mechanical constitutive modelling purposes.

![Figure 1: Reinterpretation of shear strength envelope on Canadian Glacial Till (test data from Vanapalli et al., 1996)](image)

**References**


